Minitab

ACTIVITY SHEET: NORMAL DISTRIBUTION
This activity sheet includes exercises to assess students' understanding of important concepts presented in the *Normal Distribution* lesson.

Normal Distribution

A data set is not provided for these exercises. The standard normal table is provided at the end of this activity sheet.

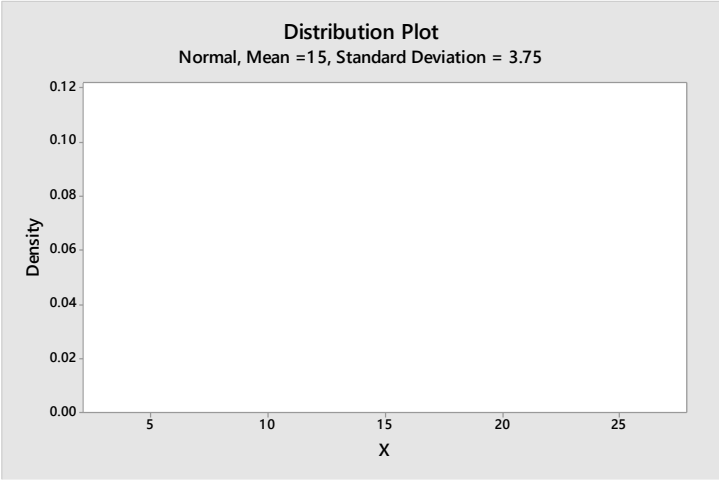
Exercise 1

The Normal Distribution

Red Block Flight Simulator
http://members.iinet.net.au/~pontipak/redsquare.html

Let the random variable *X* be the amount of time that an individual is able to "survive" in the **Red Block Flight Simulator**. It is known that *X* follows a normal distribution with $\mu =$ 15 seconds and $s = 3.75$ seconds

(a) Sketch (as best you can) the normal curve with μ = 15 seconds and σ = 3.75 seconds.



There's a *very* easy way to construct this graph in Minitab.

Minitab desktop (20 or higher)

1 Choose **Graph > Probability Distribution Plot**.
2 Choose **View Single**, then click **OK**.
3 From **Distribution**, choose **Normal**.
4 In **Mean**, type *15*.
5 In **Standard Deviation**, type *3.75*.
6 Click **OK**.

Minitab web app

1 Choose **Graph > Probability Distribution Plot**.
2 Choose **View Distribution**, then click **OK**.
3 From **Distribution**, choose **Normal**.
4 In **Mean**, type *15*.
5 In **Standard Deviation**, type *3.75*.
6 Click **OK**.

(b) Take at most 10 test "flights" at http://members.iinet.net.au/~pontipak/redsquare.html, and record your survival times. Of the flights, what is your longest survival time (in seconds)?

(c) What proportion of survival times, according to the normal distribution with μ = 15 seconds and σ = 3.75 seconds, is longer than your best survival time? Hint: Convert your best survival time to a *z*-score and use the normal table to calculate the answer.

(d) What proportion of survival times is shorter than your best survival time?

We can use Minitab to compute the probabilities in parts **(c)** and **(d)**.

Minitab desktop (20 or higher)

1 Choose **Graph > Probability Distribution Plot**.
2 Choose **View Probability**, then click **OK**.
3 From **Distribution**, choose **Normal**.
4 In **Mean**, type *15*.
5 In **Standard Deviation**, type *3.75*.
6 Click the **Shaded Area** tab. Under **Define Shaded Area By**, choose **X Value**.
7 Click **Right Tail**, since we want the proportion of survival times greater than your best time for part **(c)**. In **X value**, type your best survival time, e.g. *13.14*.
8 Click **OK**.

Minitab web app

1 Choose **Graph > Probability Distribution Plot**.

2 Choose **View Probability**, then click **OK**.
3 From **Distribution**, choose **Normal**.
4 In **Mean**, type *100*.
5 In **Standard Deviation**, type *10*.
6 Click **Options** and choose **A specified x value.**
7 Click **Right Tail**, since we want the proportion of survival times greater than your best time for part **(c)**. In **X value**, type your best survival time, e.g. *13.14*.
8 Click **OK**.

(e) I'm taking applications for my Halo 3 team, but I only want to interview applicants whose flight survival times are in the top 1%. What should I set as the minimum survival time necessary for consideration? First do this by-hand using the standard normal table. Hint: Determine the *z*-score corresponding to the top 0.01 area in the right tail of the standard normal curve. Then back transform the *z*-score into the original *x* scale, flight time in seconds.

You can verify your answer using Minitab:

Minitab desktop (20 or higher)

1 Choose **Graph > Probability Distribution Plot**.
2 Choose **View Probability**, then click **OK**.
3 From **Distribution**, choose **Normal**.
4 In **Mean**, type *15*.
5 In **Standard Deviation**, type *3.75*.
6 Click the **Shaded Area** tab. Under **Define Shaded Area By**, choose **Probability**.
7 Click **Right Tail**, since we want the *x* value corresponding to 0.01 in right tail. In **Probability**, type *0.01*.
8 Click **OK**.

Minitab web app

1 Choose **Graph > Probability Distribution Plot**.
2 Choose **View Probability**, then click **OK**.
3 From **Distribution**, choose **Normal**.
4 In **Mean**, type *15*.
5 In **Standard Deviation**, type *3.75*.
6 Click **Options** and choose **A specified probability**.
7 Click **Right Tail**, since we want the *x* value corresponding to 0.01 in right tail. In **Probability**, type *0.01*.
8 Click **OK**.

(f) Based on your best survival time, could you apply for my Halo 3 team? Why or why not?

Exercise 2

The following letter appeared in the popular "Dear Abby" newspaper advice column in the 1970s:

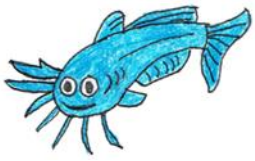
> ***Dear Abby:*** *You wrote in your column that a woman is pregnant for 266 days. Who said so? I carried my baby for ten months and five days, and there is no doubt about it because I know the exact date my baby was conceived. My husband is in the Navy and it couldn't have possibly been conceived any other time because I saw him only once for an hour, and I didn't see him again until the day before the baby was born.*
>
> *I don't drink or run around, and there is no way this baby isn't his, so please print a retraction about the 266-day carrying time because otherwise I am in a lot of trouble.*
>
> *San Diego Reader*

Whether or not San Diego Reader is telling the truth is a judgment that lies beyond the scope of this exercise, but quantifying the plausibility of her story does not. According to the collective experience of generations of pediatricians, pregnancy durations, let's call them X, tend to be normally distributed with μ = 266 days and σ = 16 days. Perform a probability calculation that addresses San Diego Reader's credibility, presuming she was pregnant for 308 days. What would you conclude and why? Show the work behind your answer, including a z-score and probability.

Exercise 3

Tall Fish Tales Inn serves fresh catfish filets at their all-you-can-eat Friday fish fry. Catfish filets served at the Inn have weights that are normally distributed with mean μ = 6.3 oz. and standard deviation σ = 0.04 oz.

(a) The Inn receives customer complaints for serving "tiny" filets, where tiny qualifies as a filet weighing less than 6.21 oz. Determine a tiny filet's z-score and the probability that a randomly selected customer will be served one. Construct a Minitab plot showing the desired probability.



(b) A waiter at the Inn has been serving all day and wants to leave for the evening with a good tip. He randomly selects filets in hopes of getting a jumbo one, where jumbo qualifies as a filet weighing more than 6.4 oz. Determine a jumbo filet's z-score and the probability that a randomly selected filet will turn out to be jumbo. Construct a Minitab plot showing the desired probability.

(c) Determine the filet weight at which only 5% of filets weigh less than it. Use the normal table and show your calculations. Construct a Minitab plot showing the desired weight.

Exercise 4

Let X represent the duration of an eight-week-old baby's smile, where X is normally distributed with mean μ = 9.1 seconds and standard deviation σ = 2.2 seconds.

(a) What is the probability that a randomly chosen eight-week-old baby smiles between 6 and 12 seconds? Compute the answer by hand and check it with Minitab using the process described below.

Minitab desktop (20 or higher)

1 Choose **Graph > Probability Distribution Plot**.
2 Choose **View Probability**, then click **OK**.
3 From **Distribution**, choose **Normal**.
4 In **Mean**, type *15*.
5 In **Standard Deviation**, type *3.75*.
6 Click the **Shaded Area** tab. Under **Define Shaded Area By**, choose **X Value**.
7 Click **Middle**, since we want the proportion of eight-week-old babies with a smile duration between two values. In **X value 1**, type *6*. In **X value 2** type *12*.
8 Click **OK**.

Minitab web app

1 Choose **Graph > Probability Distribution Plot**.
2 Choose **View Probability**, then click **OK**.
3 From **Distribution**, choose **Normal**.
4 In **Mean**, type *15*.
5 In **Standard Deviation**, type *3.75*.
6 Click **Options** and choose **A specified x value**.
7 Click **Middle**, since we want the proportion of eight-week-old babies with a smile duration between two values. In **X value 1**, type *6*. In **X value 2** type *12*.
8 Click **OK**.

(b) Determine the 90th percentile for the duration of an eight-week-old baby's smile. That is, find the value x such that the probability of being less than or equal to x is 0.90. Set up the correct expression and show your work. Check your answer with Minitab.

Exercise 5

Every night when you get home from school, you take your dog Aiko for a walk. The length of the walk is normally distributed with a mean of $\mu = 12$ minutes and standard deviation of $s = 4$ minutes.

(a) What proportion of walks last longer than 20 minutes? Answer to four decimal places.

(b) What proportion of walks last between 10 and 16 minutes? Answer to four decimal places.

Exercise 6

The Salty Spud Potato Chip Company uses a machine to fill 8 oz. bags with Super Salty chips. Because of irregularities in the chips, there is some variability in the actual amount of chips that the machine puts into each of the bags. The amount of chips put into the bags can be adequately modelled with a normal distribution with μ = 7.98 oz. and σ = 0.012 oz. You may solve this problem using a *z* table or Minitab.

(a) What proportion of bags filled by this machine will contain less than 8 oz. of chips? Show your work.

(b) What amount of chips (in ounces) is exceeded only 2.5% of the time? Show your work. Answer to 3 or 4 decimal places.

Exercise 7

You played sports throughout high school and college, and your knees have taken a hit. Your mom has had knee surgery, and you hope you don't need it as well! Here's a little information about knee surgeries:

An article in *Knee Surgery, Sports Traumatology, Arthroscopy* ["Effect of Provider Volume on Resource Utilization for Surgical Procedures" (2005, Vol. 13, pp. 273–279)] showed a mean time of 129 minutes and a standard deviation of 14 minutes for anterior cruciate ligament (ACL) reconstruction surgery at high-volume hospitals (with more than 300 such surgeries per year).

(a) What is the probability that a randomly selected ACL surgery at a high-volume hospital exceeds a time that is more than two standard deviations above the mean? Show your work, either by hand or in Minitab.

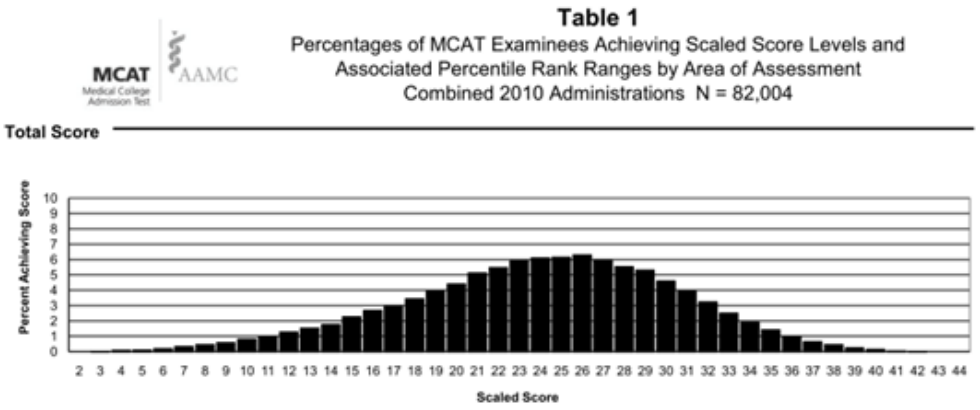
(b) What is the probability that your ACL surgery at a high-volume hospital is completed in less than 100 minutes? Show your work, either by hand or in Minitab.

(c) What surgery time (in minutes) is exceeded only 5% of the time? Show your work, either by hand or using Minitab.

Exercise 8

The distribution of the 2010 overall MCAT scores is given in the table below. To get into Harvard, you need to have a score above 36. What percentile does this score represent? Note: The p^{th} percentile has p percent of scores less than it. Show your work, either by hand or using Minitab.

Table Information: Mean = 25, Standard Deviation = 6.4, Maximum = 44



Exercise 9

Adult men's heights (in inches) are normally distributed with $\mu = 70$ and $\sigma = 3$. Adult women's heights (in inches) are normally distributed with $\mu = 67$ and $\sigma = 2$. Members of the "Beanstalk Club" must be at least six feet (72 inches) tall. Show your work, either by hand or using Minitab.

(a) If a man is selected at random, what is the probability that he is eligible for the Beanstalk Club?

(b) If a woman is selected at random, what is the probability that she is eligible for the Beanstalk Club?

(c) [BONUS] If a man and a woman are selected at random from their respective populations, what is the probability that the pair is eligible for the Beanstalk Club? Assume that we pick the man and woman independently; for example, the man and woman are not a couple or related to each other. Answer to 4 decimal places.

Exercise 10

The grades for a certain exam are normally distributed with mean 67 and variance 64.

(a) What percent of students get B's (80-90)?

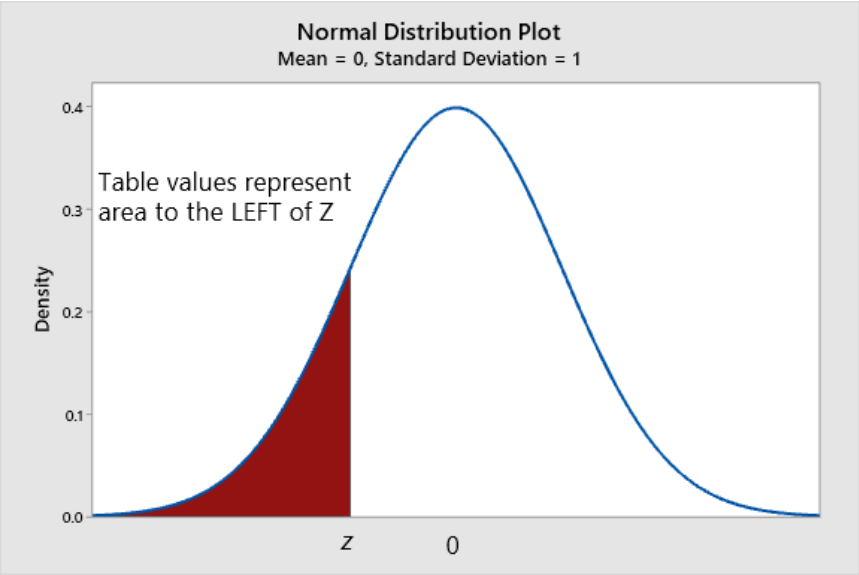
(b) What percent of students get F's (< 60)?

Exercise 11

Manufactured parts have lifetimes in hours, X, that are normally distributed with mean 1000 and standard deviation 100. If $800 \leq X \leq 1200$, the manufacturer makes a profit of \$50 per part. If $X > 1200$, the profit per part is \$75. Otherwise, the manufacturer loses \$25 per part. What is the expected profit per part?

Using the Standard Normal Distribution Table

The graph below depicts how to interpret the standard normal distribution tables provided on the following pages.



STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
-0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997