Minitab

ACTIVITY SHEET: POPULATION MEAN HYPOTHESIS TESTING FOR LARGE SAMPLES

This activity sheet includes exercises to assess students' understanding of important concepts presented in the *Population Mean Hypothesis Testing for Large Samples* lesson.

Population Mean Hypothesis Testing for Large Samples

The data for these exercises are in the Minitab file ***HypTestForMean_LargeSample_Activity.mtw***.

Exercise 1

For the following multiple choice problems, choose the best answer.

(a) The test statistic and p-value corresponding to the hypothesis test H_0: $\mu = 2$ versus H_1: $\mu > 2$ are $\bar{x} = 2.076$ and $p = 0.065$, respectively. Which of the following is an appropriate interpretation of this p-value?

A. The probability that the null hypothesis is true is 0.065.

B. The probability that the alternative hypothesis is true is 0.065.

C. The data is not normally distributed.

D. In repeated sampling, the probability of observing a test statistic at least as extreme as $\bar{x} = 2.076$ is 0.065.

(b) A pudding manufacturer packages its product into bags weighing 1 kilogram, on average. The manufacturer's statistician has discovered that the setting of the machine is causing the fill weights to drift. The statistician needs to detect shifts in the mean weight as quickly as possible and reset the machine when appropriate. In order to detect shifts in the mean weight, he collects a random sample of 50 bags periodically, weighs them, and calculates the mean and standard deviation. The data from this afternoon's sample yields a sample mean of 1.03 kg. The population standard deviation is 0.08 kg. Determine the p-value for the hypothesis test H_0: $\mu = 1$ versus H_1: $\mu \neq 1$.

A. 0.008

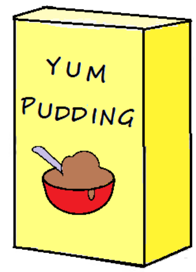
B. 0.004

C. 0.011

D. 0.005

E. 0.704

F. 0.996



G. We cannot determine the *p*-value because the data does not come from a normal distribution.

We can calculate the *p*-value in Minitab:

1 Open the 1-sample Z dialog box. Choose **Stat > Basic Statistics > 1-Sample Z**.
2 From the drop-down list, select **Summarized data**.
3 In **Sample size**, enter *50*.
4 In **Sample mean**, enter *1.03*.
5 In **Known standard deviation**, enter *0.08*.
6 Select **Perform hypothesis test**. In **Hypothesized mean**, enter *1*.
7 Click **OK**.

(c) The pain reliever currently used at General Hospital brings relief to patients in a mean time of 3.5 minutes. To compare a new pain reliever with the current one, the new pain reliever is administered to a random sample of n = 50 patients. The mean time to feel relief for this sample of patients is 2.8 minutes. The population standard deviation is 1.14 minutes.

What are the appropriate null and alternative hypotheses to determine if the mean time for patients to feel relief from the new pain reliever is less than the time required for the hospital's current pain reliever?

A. $H_0: \mu = 2.8$ versus $H_1: \mu < 2.8$

B. $H_0: \bar{x} = 2.8$ versus $H_1: \bar{x} < 2.8$

C. $H_0: \mu = 2.8$ versus $H_1: \mu > 2.8$

D. $H_0: \mu = 3.5$ versus $H_1: \mu < 3.5$

E. $H_0: \bar{x} = 3.5$ versus $H_1: \bar{x} < 3.5$

F. $H_0: \mu = 3.5$ versus $H_1: \mu > 3.5$

(d) We are testing $H_0: \mu = 2$ versus $H_1: \mu > 2$ and conclude that we can reject the null hypothesis H_0 at significance level $\alpha = 0.05$. Suppose we decide to change the alternative hypothesis from $H_1: \mu > 2$ to $H_1: \mu \neq 2$. Using the same data, can we still reject H_0 at $\alpha = 0.05$?

A. Yes

B. No

C. There is not enough information to answer this question.

(e) We are conducting the following hypothesis test: H_0: $\mu = 9.5$ versus H_1: $\mu > 9.5$. The Minitab output for this 1-sample Z-test is provided below.

Descriptive Statistics

N	Mean	SE Mean
50	9.728	0.122

μ: mean of Sample
Known standard deviation = 0.86

Test

Null hypothesis	H_0: $\mu = 9.5$
Alternative hypothesis	H_1: $\mu > 9.5$

The approximate *p*-value for this hypothesis test is:

A. 0.015

B. 0.031

C. 0.058

D. 0.267

E. 0.395

F. 0.971

G. We cannot determine the *p*-value because we don't know if the data is from a normal distribution.

(f) A certain brand of orange juice is advertised to contain 85% fruit juice per bottle. A random sample of 32 bottles of this juice is selected in order to perform the hypothesis test:

$$H_0: \mu = 0.85 \quad \text{versus} \quad H_1: \_\_\_\_\_,$$

where μ represents the average percentage of juice per bottle. The Minitab output for the test is:

Descriptive Statistics

N	Mean	SE Mean	Z-Value	P-Value
32	0.8228	0.0335	-0.81	0.417

What is the correct alternative hypothesis for this test?

A. H_1: $\mu < 0.82$

B. H_1: $\mu < 0.85$

C. H_1: $\mu > 0.82$

D. H_1: $\mu > 0.85$

E. H_1: $\mu \neq 0.82$

F. H_1: $\mu \neq 0.85$

(g) A quality control specialist takes several measurements to test H_0: $\mu = 2$ versus H_1: $\mu \neq 2$. She computes a *p*-value of 0.01 for the hypothesis test. Which interpretation is correct?

A. The probability that $\mu = 2$ is 0.01

B. The probability that $\mu \neq 2$ is 0.01.

C. The probability that the quality control specialist conducted the study properly is 0.99.

D. The probability of the quality control specialist observing those results (or more extreme) if $\mu = 2$ is 0.01.

(h) Which of the following statements is <u>not</u> true regarding hypothesis testing?

A. The alternative hypothesis is the assertion that is contradictory to the null hypothesis.

B. The null hypothesis is the claim that is assumed to be true, or the "status quo" hypothesis.

C. The sample evidence is used to determine whether or not to reject the alternative hypothesis.

D. The two possible conclusions from a hypothesis test are reject H_0 or fail to reject H_0.

E. When drawing a conclusion after performing a hypothesis test, there is always a chance that you made the wrong decision, even if that chance is very small.

(i) In performing the hypothesis test H_0: $\mu = 8$ versus H_1: $\mu \neq 8$, the resulting *p*-value is 0.016. Thus, if we construct a 95% confidence interval for μ using the exact same data, $\mu = 8$ will not be included in the 95% confidence interval.

A. True

B. False

C. We do not have enough information to determine this.

(j) You perform the hypothesis test H_0: μ = 180 versus H_1: μ > 180 for the average number of minutes per day that students at your school watch TV. Which of the following statements is correct regarding the *p*-value?

A. Assuming the null hypothesis is true, an extremely small *p*-value indicates that the sample mean calculated from the sample data is extremely different from null mean μ = 180.

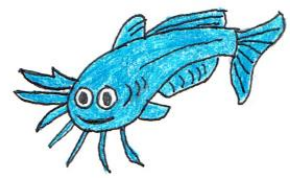
B. The *p*-value measures the probability that the alternative hypothesis is true.

C. The *p*-value measures the probability that the null hypothesis is true.

D. The larger the *p*-value, the stronger the evidence against the null hypothesis.

E. A large *p*-value indicates that the data supports the alternative hypothesis.

(k) Under normal environmental conditions, adult catfish in Dog Lake have an average length of μ = 13.9 cm with a population standard deviation σ = 2.1 cm. Students who frequently fish at Dog Lake claim that the catfish are smaller than usual this year. Suppose your statistics class takes a random sample of adult catfish from Dog Lake. Which of the following provides the **strongest** evidence to support the claim that students are catching smaller than average length (13.9 cm) catfish this year?



A. A random sample of size n = 36 with a sample mean of $\bar{x} = 13.5$ cm.

B. A random sample of size n = 36 with a sample mean of $\bar{x} = 13.3$ cm.

C. A random sample of size n = 121 with a sample mean of $\bar{x} = 13.5$ cm.

D. A random sample of size n = 121 with a sample mean of $\bar{x} = 14.5$ inches.

E. We do not have enough information to determine this.

(l) You just bought a new laptop with a supposed true population mean battery life of 3 hours. You survey friends who have the same laptop to determine if, in fact, the true population mean is less than 3 hours. For a random sample of n = 40 friends, you calculate a sample mean of 3.25 hours. To test the "less than" speculation, you should use the following hypothesis:

$$H_0: \mu = 3 \quad \text{versus} \quad H_1: \mu < 3$$

A. True

B. False

C. We do not have enough information to determine this.

(m) The *Central Limit Theorem*, the key to performing a hypothesis test for the population mean μ with a *Z* test, states that:

A. We can always use a normal curve to approximate the distribution of the sample mean $\bar{X}$.

B. If *n* is large (e.g. *n* > 30) and the original population distribution is normal, then the distribution of the sample mean $\bar{X}$ can be approximated by a normal curve.

C. We can always use a normal curve to approximate the distribution of a random variable *X*.

D. If *n* is large (e.g. *n* > 30) then the distribution of the sample mean $\bar{X}$ can be approximated closely by a normal curve even if the original distribution is not normal.

E. If *n* is large (e.g. *n* > 30), then the distribution of the random variable *X* can be approximated closely by a normal curve even if the original distribution is not normal.

Exercise 2

Putting Puzzles Together

If students are helping to conduct this exercise in class, at least 30 puzzle times are needed to be able to apply the concepts, namely a 1-sample *Z* hypothesis test for the population mean. Instructions for creating the data set are below. If you do not have time or puzzles to collect class data or do not have at least 30 students to perform this activity, then you can use the 'Puzzle Times (secs)' data found in ***HypTestForMean_LargeSample_Activity.mtw***. For this data, the population mean is $\boldsymbol{\mu_0 = 90}$ with sample size $\boldsymbol{n = 43}$.

Activity Instructions: To obtain data for this exercise, students will need easy jigsaw puzzles – all identical – consisting of 25 to 40 pieces, such as the ones found at dollar stores. Alternatively, you can find free online puzzle websites such as www.jigzone.com.

At the beginning of class, hand out the puzzles to the students. You can have groups of students share one puzzle and take turns. Tell them to record (in seconds) the amount of time it takes to complete the puzzle. Other group members should not watch each other to avoid biasing the results. Times can be recorded using a cell phone. Alternatively, have students bring their laptops to class and send them the link to a given online puzzle, which typically measures the time for you.

As students finish the puzzle, ask them to go to the board to record their puzzle construction times. Once everyone has recorded his or her data, the instructor can enter the data in Minitab.

The instructor can then email the worksheet data to students to ensure that everyone is working with the same data set. Alternatively, the instructor can use a Google spreadsheet.

Activity "Story": At a circuit board company, employees are required to assemble circuit boards (puzzles) correctly and completely in a desired amount of time. If they are too slow or inaccurate, then the entire assembly line is affected.

Based on former employee records, the company states that these circuit boards can be assembled in $\mu_0 = 90$ seconds. [Instructors may need to provide a different μ_0 based on their students' puzzle times.]

You are the manager of these employees. You suspect that the true mean time is not accurate. Specifically, you think the true mean time μ to assemble these boards is more than 90 seconds, and employees are being unfairly assessed. In order to test the hypothesis:

$$H_0: \mu = \mu_0 \text{ seconds} \qquad \text{versus} \qquad H_1: \mu > \mu_0 \text{ seconds}$$

you randomly select *n* workers to assemble these boards (puzzles). Their average time to assemble the boards is $\bar{x}$ seconds with standard deviation *s* seconds. [Instructors will need to provide values for *n*, $\bar{x}$, and *s* based on the class's data. The data provided in the Minitab worksheet has $n = 43$ and $\bar{x} = 95.65$ seconds. The population standard deviation is $\sigma = 28$ seconds.

(a) Can we assume that the distribution of the mean time $\bar{X}$ for $n = 43$ employees is normally distributed? Why or why not? Be specific.

(b) Determine the standardized test statistic *z* and its *p*-value by hand.

(c) Determine the standardized test statistic *z* and its *p*-value in Minitab.

1. Open the 1-sample Z dialog box. Choose **Stat > Basic Statistics > 1-Sample Z**.
2. In **One or more samples, each in a column**, enter *Puzzle Times (secs)*.
3. In **Known standard deviation**, enter *28*.
4. Select **Perform hypothesis test**. In **Hypothesized mean**, enter *90*.
5. Select **Options**.
6. In **Alternative hypothesis**, choose **Mean > hypothesized mean**.
7. Click **OK** in each dialog.

(d) Can we reject H_0 at an $\alpha = 0.10$ level of significance? Why or why not?

(e) Can we reject H_0 at an $\alpha = 0.05$ level of significance? Why or why not?

Exercise 3

Cutting Paper Strips

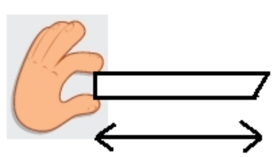
Reference: "The Blind Paper Cutter: Teaching about Variation, Bias, Stability, and Process Control," by Richard A. Stone (August 1998). *The American Statistician*, 52(3), pp. 244-247.

If you do not have the time or materials required to collect class data, then you can use the 'Cut Lengths (cm)' data found in ***HypTestForMean_LargeSample_Activity.mtw***. For this data, the mean is ***μ_0 = 14 cm*** (half the length of the 28 cm paper strips). There are only 25 lengths in the dataset, but the data in that column is normally distributed.

Activity Instructions: To obtain data for this exercise, groups of 3-4 students will cut 30 (or more) equal length strips of paper. For example, you can use 8.5 x 11 inch scrap paper to create the strips. It doesn't matter how long the strips are as long as they are all the ***same length***. The mean μ_0 for the questions below is equal to ½ of the length.

It is recommended that the instructor or students create the strips of paper prior to class. During class, each group will need a ruler, as well as a pair of scissors to cut the paper. Each group will need to select one person to be the cutter, and then do the following:

- Before cutting the strips, measure the length. Every strip should be the same length. Then divide the length by 2 to calculate the value of μ_0. For example, if the strips are 28cm, then μ_0 = 14 cm.
- Cutter: Cut ~30 strips of paper ***in half*** from the given pile of strips in front of you. Keep your ***eyes closed*** for the entire experiment, although you can use your scissors to "feel" along the length of the strip.
- Other group members: Presuming the cutter is right-handed, take each ***strip remaining in the cutter's left hand*** and measure the ***minimum length***. Clean up the fallen strips to keep them out of the cutter's way as you go. The fallen strips will *not* be measured.



Measure shortest length distance of strip in left hand

- Enter the minimum strip lengths into Minitab and email the data to your instructor.

Perform the following three hypothesis tests in Minitab, using a population standard deviation of σ = 1.2. What conclusions can you draw from the *p*-values reported for each hypothesis test?

(a) H_0: $\mu = \mu_0$ versus H_1: $\mu \neq \mu_0$

(b) H_0: $\mu = \mu_0$ versus H_1: $\mu < \mu_0$

(c) H_0: $\mu = \mu_0$ versus H_1: $\mu > \mu_0$

Exercise 4

A drug manufacturer claims a given type of medicine contains 2.5 milligrams of a certain active ingredient per capsule. An independent laboratory takes a random sample of 20 of these capsules and measures the amount of the active ingredient in each. The distribution of active ingredients per capsule is normally distributed.

3.31	1.94	2.97	3.15	1.94	1.30	2.23	2.91	2.54	0.88
0.61	2.35	1.70	1.84	0.83	2.42	0.96	2.05	2.23	1.92

The laboratory has been hired to determine if the true mean amount of the active ingredient is actually less than 2.5.

(a) Using the proper statistical notation, write down the null and alternative hypotheses.

H_0:______________ versus H_1: ______________

(b) What does the parameter of interest μ represent in words? Select the best answer.

A. The true mean amount of the ingredient in all capsules of this type.

B. The true mean amount of the ingredient in the 20 capsules of the laboratory's sample.

C. The difference between the true mean amount of the ingredient in the population of all capsules and the sample mean amount of the ingredient in the 20 capsules in our sample.

D. The true proportion of all capsules of this type that contain less than 2.5 milligrams of the ingredient.

E. The amount of the ingredient in a randomly selected capsule of this type.

(c) Using Minitab, determine the mean and standard deviation of the sample data provided. The data is in the Minitab column "Active Ingredient."

(d) Assume that the amount of the ingredient in capsules is normally distributed. Also, assume that the laboratory is told that the population standard deviation is $\sigma = 0.8$ mg. By hand, calculate the z statistic and p-value for the hypothesis test in part **(a)** based on the sample data provided.

(e) Verify the z statistic and p-value using Minitab. From the 1-Sample Z dialog box, click **Options** to select the appropriate alternative hypothesis.

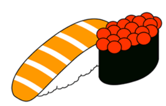
Exercise 5

A waiter (with a statistics background) who works at the "World of Sushi" restaurant is asked to estimate the true mean time to clean and set tables on a busy Saturday night. This knowledge will help the restaurant owners determine how much time they should schedule between reservations.

Suppose the waiter randomly samples times for $n = 40$ tables on busy Saturday nights and obtains a sample mean of $\bar{x} = 4.2$ minutes. The population standard deviation is $\sigma = 0.8$ minutes. The sample data is used to construct the following 95% confidence interval for the true mean clean-up time μ:

[3.952, 4.448]

Based on the 95% confidence interval, can the null hypothesis for the following hypothesis test be rejected at an $\alpha = 0.05$ level of significance? Explain why or why not.



$$H_0: \mu = 3.5 \text{ minutes} \quad \text{versus} \quad H_1: \mu \neq 3.5 \text{ minutes}$$

Exercise 6

A random sample of $n = 50$ drill bits is used to put holes into a steel doorframe. The lifetime of a drill bit is measured as the number of holes drilled before the bit fails. The average lifetime of a drill bit is 12.68 holes. The population standard deviation is 6.83 holes. Calculate the z statistic and p-value for the following hypothesis tests by hand or in Minitab.

(a) $H_0: \mu = 12$ holes versus $H_1: \mu > 12$ holes

(b) H_0: μ = 12 holes versus H_1: $\mu \neq$ 12 holes

(c) H_0: μ = 12 holes versus H_1: μ < 12 holes

Exercise 7

According to a Google search, the average height of male soccer players in the U.S. is normally distributed with mean 1.79 m with a standard deviation of 0.04 m.

(a) For a randomly selected soccer team of 11 players, what is the probability that the *average* height of the players is less than 1.77 m? Calculate the probability by hand or in Minitab.

(b) In view of the small sample size, must you make any additional assumptions to justify the answer to part **(a)**? Please provide a short explanation.

(c) Set up the hypothesis test to test the true average height of male U.S. soccer players.

H_0:______________ **versus** **H_1:** ______________

(d) Determine the *z* statistic and *p*-value for the hypothesis test in part **(c)**.

Exercise 8

According to a study, the U.S. mean family income is $63,091 and a standard deviation of $21,000.

(a) If a consulting agency surveys 49 families at random, what is the probability that it finds a *mean* family income that is more than $71,500? Calculate the probability by hand or in Minitab.

(b) For the hypothesis test H_0: μ = 63,091 versus H_1: $\mu \neq$ 63,091, determine the *z* statistic and *p*-value.

(c) Suppose the median income in the U.S. was $55,000. Why do many websites use the median as their indicator of income level instead of the mean?

Exercise 9

Your neighbor grows and sells cucumbers in the summer. She packages them in plastic storage bags and claims that the true mean weight of one of these bags is 1 pound. To test her claim, you take a random sample of 64 of these cucumber-filled bags, weigh them, and record their weights in the Minitab column "Bag Weights (lbs)." Assume the population standard deviation is 0.52 lbs.

With 90% confidence, is her claim "the mean weight of these bags is 1 pound" true? Use either a hypothesis test or confidence interval to provide supporting evidence for or against this claim. The descriptive statistics for the data are provided below.

Statistics

Variable	Total Count	Mean	Minimum	Median	Maximum
Bag Weights (lbs)	64	0.8771	-0.2309	0.8638	2.0413